$d-$dimensional array

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All symbols stand for nonnegative integers.

Let $(n_1, n_2, \ldots, n_d)$ be the dimensions of the array, size of the array is $N = n_1 n_2 \ldots n_d$ elements.

An element of the array has index $(i_1, i_2, \ldots, i_d)$, $0 \le i_k < n_k$, $1 \le k \le d$.

Elements of the array are stored in the memory in the lexicographic order: $(i_1, i_2, ..., i_d) \le (j_1, j_2, ..., j_d)$ if and only if $i_k \le j_k$ for some k, $1 \le k \le d$ and $i_l = j_l$ for $l > k$, and the position of the element with index $(i_1, i_2, \ldots, i_d)$ is $m\,(i_1, i_2, ..., i_d) = i_1 + n_1\,(i_2 + n_2\,(i_3 \ldots n_{d-1} i_d) \ldots)$.

Let $0 \le k \le d$: i) the set of the elements with indices $(i_1, i_2, \ldots, i_d)$ such that $0 \le i_k < n_k$, i_l fixed for $l \ne k$, is called row. ii) the set of the elements with indices $(i_1, i_2, \ldots, i_d)$ such that i_k fixed, $0 \le i_l < n_l$ for $l \ne k$, is called slice.

Hence row is a $1-$dimensional orthogonal section of the array, slice is a $d-1-$dimensional orthogonal section of the array.

How to traverse row: $for(i = 0; i < num; i++)$

$//$ process element with position $first + i \cdot step$

where

$first = m\,(i_1, i_2, ..., i_d)$ with $i_k = 0$, $step = n_1 n_2 \ldots n_{k-1}$, $num = n_k$

How to traverse slice: $for(ind = first; ind < totnum; ind+ = step)$

$//$ process contiguous interval of elements of length $runlen$ starting at position ind

where

$first = n_1 n_2 \ldots n_{k-1} i_k$, $runlen = n_1 n_2 \ldots n_{k-1}$, $step = n_1 n_2 \ldots n_k$, $num = n_1 n_2 \ldots n_{k-1} n_{k+1} \ldots n_d$, $totnum = num \cdot step$

Traversing all rows in the $k^{th}-$direction: iterate $first$ in the row traversal algorithm over slice with $i_k = 0$.

Traversing all rows in all directions: iterate k over all dimensions.